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ANALYSIS OF TOPOLOGICAL SPACES OF THE SET OF GRAPHS

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ABSTRACT

Now-a-days the word 'topology' is being commonly used and getting popularity day by day in the filed of modern mathematics while in the last century when India failed in her trial of being free the British rule, the word topology was found on the tongue of rare mathematician. *The word topology seems to be derived from Greek wors :* 'topo' means 'a place' and 'logo' means 'a discourse'. The meaning of topology is 'by associating the things with particular place or Town.

Key words: Modern Mathematics, topology

TOPOLOGY

Given a non empty set X i.e. X, a class $\Im$ of subsets of X is said to be a topology on X is $\Im$ satisfies the following axiones:

(i) $\Im_1 : \phi, X \varepsilon \Im$

(ii) $\Im_2$: arbitrary union of sets in $\Im$ are in *i.e.*

$A_\alpha \in \Im, \forall\alpha \in A$

where A is an arbitrary (index) set, then

$\cup \{A\alpha : \alpha \in A\} \in \Im$

(iii) $\Im_3$: finite intersection sets in $\Im$ are in $\Im$ e.g.

$A \in \Im$ and $B \in \Im$; then $A \cap B \in |\Im|$.

Let us take an example for the topology on the sets.

Examples: Let X = { a, b} and T = {ϕ, {a}, X} is a class of subsets of X. Now verity that T is topology on X.

To show that T is a topology, we have to examine the axioms of topology Since.

***$\Im_1$* is satisfied:** As the $\phi, X \in | T$

TOPOLOGY ON SET OF EDGES:

In the topology of graphs, we will consider only the simple graph. As it is defined in earlier chapter, that every simple graph has $2^{|E|}$ subgraphs, where E is equal to the total no. of edges in the simple graph.

Def: For a given non-empty edge set E of the graph G, a class T of E is said to be topology of graph on E if it satisfy the following axioms.

(i) $\Im|_1 : \phi, E \in T$

i.e. null edge set and edge set itself must lie in the class of subset T.

(ii) $\Im_2$: arbitary union of sets in $\Im|$ should lie in T

i.e. if , $E_1 \in T, \nabla\, i \in \Lambda$

where Λ is an arbitrary index set, then

$U \{E_1 : i \varepsilon \Lambda\} | T$

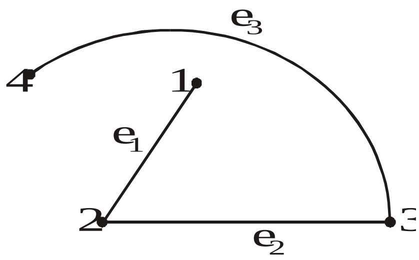
(iii) $\Im|_3$: Finite intersection of sets in T are in T

i.e. if $E_1 \in T$ and $E_j \in T$

Then $E_i \cap E_j \in T$

Above given definition can be verified on the edge set also.

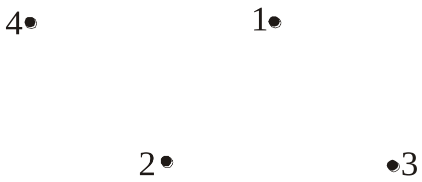
Examples: Let us consider an edge set E = [e_1, e_2, 2_3} of a simple graph G.
Let T is a class of edge subsets of E i.e.
T = { ϕ, { e_1, e_3}, E}



To verity that whether it's a topology or not we have to verify all the axioms given in the definition.

$\mathfrak{I}_1$ is satisfied: ϕ | E ε T

i.e.



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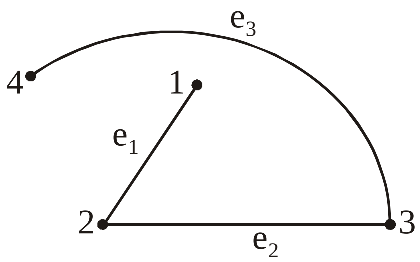
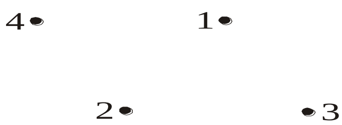


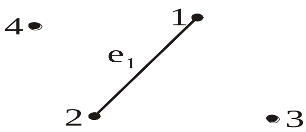
As the null edge set and edge set E itself belongs to the class of edge subset i.e. T, this axiom is satisfied.

$\mathfrak{I}_2$ is satisfied: to satisfy this axiom, it has to be shown that union of the edge subsets of the class T lies in T.

$\phi \cup \{e_1\} =$



$\cup$



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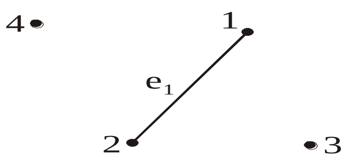
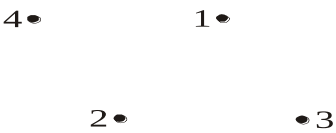


$\in T$

$\phi \cup \{e_2, e_3\} =$



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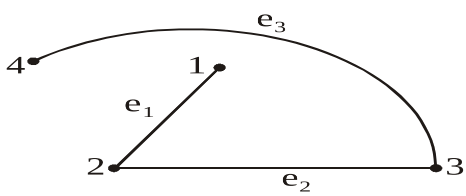
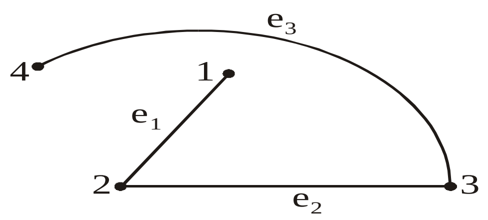
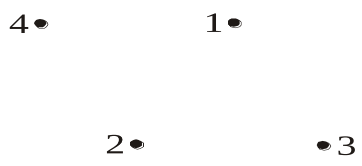
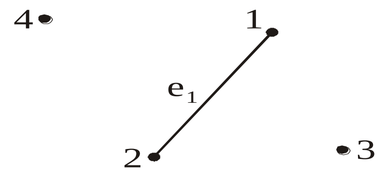


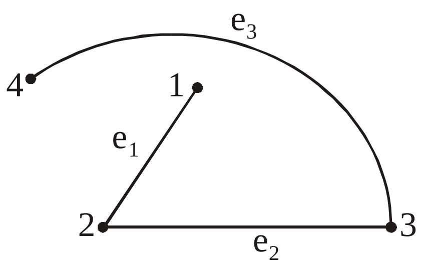
= 

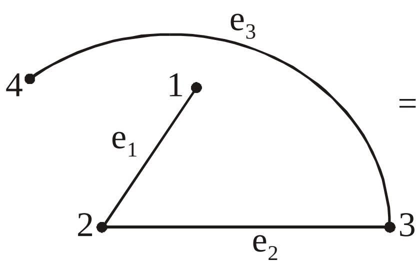
$\in$ T

$\phi \cup \{e_1\} \cup \{e_2,e_3\}$

$\phi \cup \{e_2,e_3\}$ = 

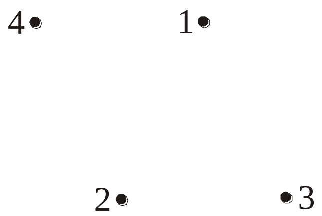
$\cup$ 

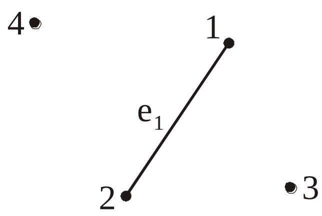
$\cup$ 

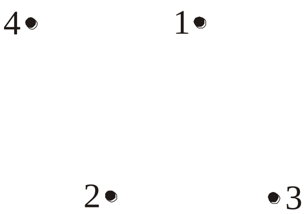
= 

= E $\in$ T

Similarly, we can say it for
$\{e_{1,}\} \cup \{e_{2,}\ e_3\}$, $\{e_{1,}\} \cup \{e_{2,}\ e_3\} \cup\ \phi \cup E$, etc.
Hence the arbitrary union of subsets lies in the class T.
$\mathfrak{I}_3$ is satisfied: This axiom is true for the class of edge subsets T, as the intersection of all the subsets of T lies in itself. Let us examine it with the help of examples :

$\phi \cap \{e_1\}$ = 

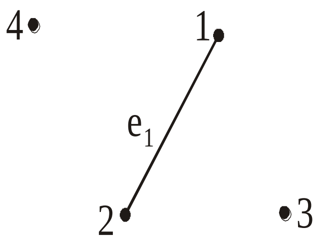
$\cap$ 

= 

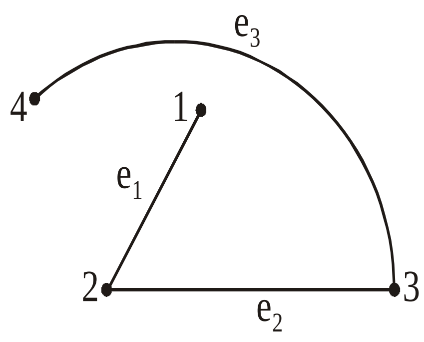
$\in$ T

Similarly the intersection of null edge set with any edge subset with any other edge subset with be a null edge set i.e. it belongs to the class of edge subset T.

$\{e_1\} \cap \{e_2, e_3\} =$



$\cap$



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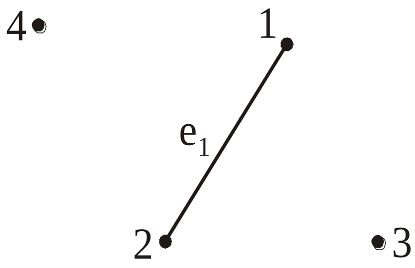
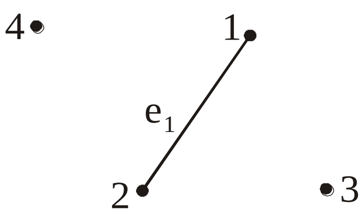


$\in T$

as no edge is common in both edge set, hence the intersection will be a null edge set.

Now

$\{e_1\} \cap E =$



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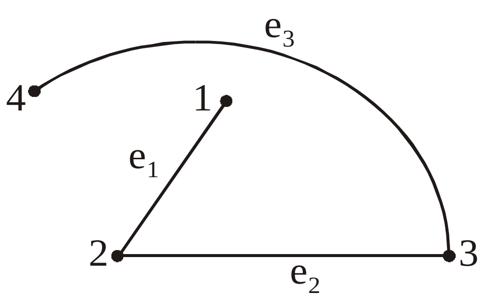

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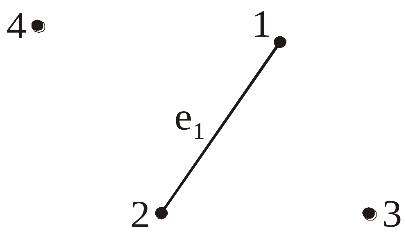
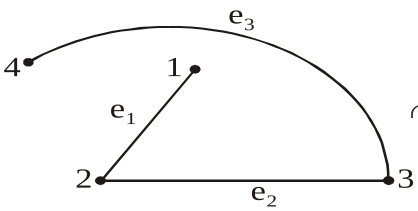


$\in T$

Similarly

$\{e_2,e_3\} \cap E =$



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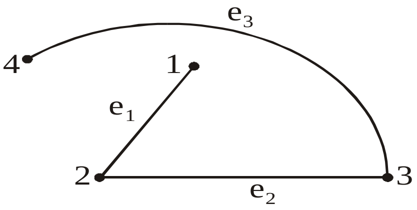

Two edges are common in both the edge set i.e. e_2 and e_3

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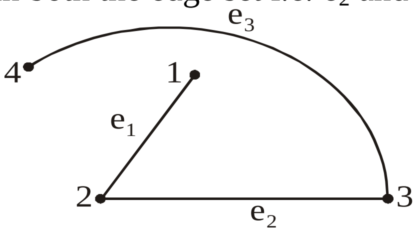


$= \{e_2,e_3\} \in T$

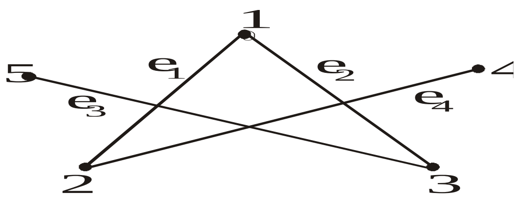
Similarly

$\phi \cap (e_1) \cap \{e_2, e_3\} \cap E = \phi \in T$

Thus the axiom of topology is satisfied by the class T on the edge set E.

Let us consider one more example for the better understanding of the concept of topology with respect to the edge set.

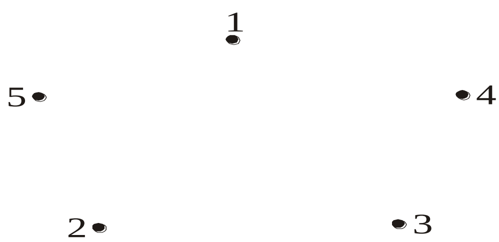
Example: Consider an edge set $E = \{e_1, e_2, e_3, e_4\}$ of a simplex graph G. Let us assume a class of edge subsets E i.e. $T = \{\phi\ E, \{e_1, e_2\}, \{e_3, e_4\}\}$.



To show that the class of subsets T is a topology, we have to satisfy the axioms of topology.

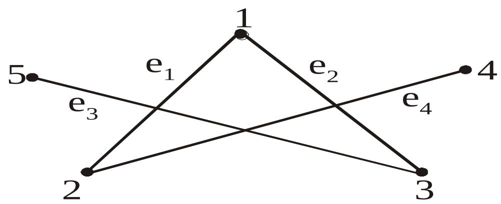
$\Im_1$ is satisfied : As $\phi, E \in T$

i.e.



$\in T$

and



$\in T$

Thus the first axiom of topology is satisfied.

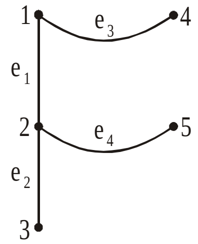
DIFFERENT TYPES OF TOPOLOGIES:

The previous section is evolved with the concept of topology on the edge set. Thus we can study the different types of topologies with respect to the set of edges. This section, will throw light on the following types of topologies: stronger and weaker, not comparable, confinite or finite complement topology, co-countable topology, intersection and union of topology, hereditary etc. Further these concept will be utilized in some theorems also.

Stronger and Weaker Topologies : Two topologies T_1 and T_2 defined on the edge set E is said to be weaker and stronger topology if $T_1 \subset | \ T_2$, then T_1 is said to weaker topology than T_2 and T_2 is said to be finer or stronger topology than T_1 let us take an example to verify the above given definition.

Example: If the edge set E = $\{e_1, e_2, e_3, e_4\}$, then define two topologies on E.

T_1 $\{\phi, \{e_1\}, E\}$ and $T_2 = \{\phi, \{e_1\}, \{e_3, e_4\}, \{e_1, e_3, e_4\}, E\}$



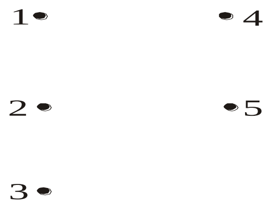
In the example the edge set E can be shown graphically :

As it is already defined that T_1 and T_2 are the topologies, thus it has to shown that either $T_1 \subset T_2$, or $T_2 \subset | \ T_1$, to say that these are weaker or stronger topologies.

$T_1 = \{\phi, E, \{e_1\}\}$, $T_2 = \{\phi, \{e_1\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}, E\}$

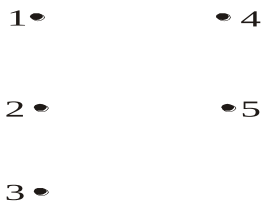
If T_1 is the subset of T_1, then every edge subset of T_1 must belong to T_2.

i.e. $\phi \in T_1$ i.e.

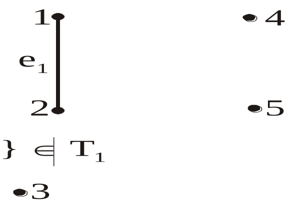


$\in T_1$

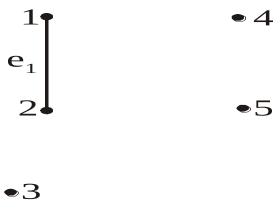
$\Rightarrow$



$\in T_2$



$T\{e_1\} \in | \ T_1$ $\in T_1$ $\Rightarrow$



$\in T_2$

also $\{e_1\} \in | \ T_1$ $\Rightarrow \{e_1\} \in T_2$

Similarly, we can observe that the subset E itself lies in T_2.

Thus we can say that T_1 is contained in T_2. Hence T_1 is a weaker topology and T_2 is stronger topology defined on the edge set E.

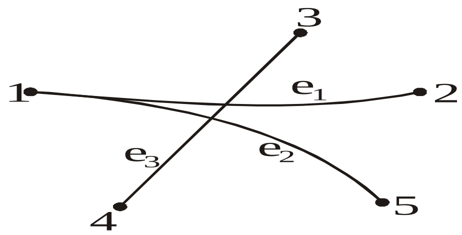
Not comparable Topologies*:* Two topologies T_1 and T_2 defined on the edge set E on the graph G is said to be noncomparable topologies if $T_1 \not\subset T_2$ and $T_2 \not\subset T_1$ i.e. T_1 is not contained in T_2 and T_2 is not contained in T_1.

1.1 let us consider an edge set E = {e_1, e_2, e_3} where T_1 and T_2 are the two topologies such that

$T_1 = \{\phi, \{e_1, e_2\}, E\}$ and $T_2 = \{\phi, \{e_1, e_3\} E\}$.

Now to show that T_1 and T_2 are noncomparable, we have to show that

$T_1 \not\subset T_2$ and $T_2 \not\subset T_1$.

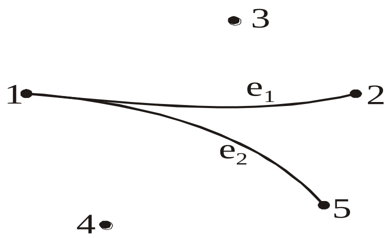


If T_1 and T_2 are not contained in each other, then all the edge subsets of T_1 and T_2 are not the number of either.

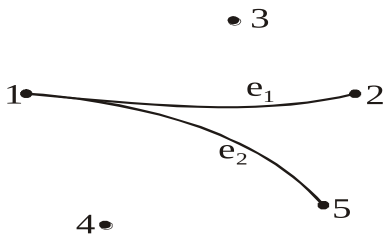
First to show $T_1 \not\subset T_2$

$\{e_1, e_2\} \phi T_1$

i.e.



$\in T_1$ but



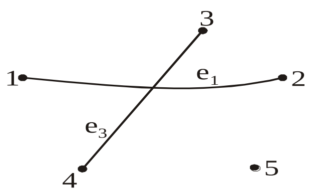
$\in T_2$

Thus all the edge subsets of T_1 does not belong to T_2.

Hence $T_1 \not\subset T_2$

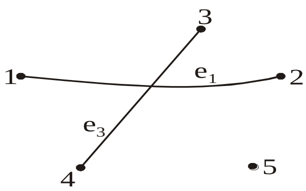
Now to verify $T_2 \not\subset T_1$

$\{e_1, e_2\} \notin T_2$ i.e.



$\in T_2$

but



$\in T_1$

Hence $T_2 \not\subset T_1$

Hence both the topologies are not contained in each other. Thus topologies T_1 and T_2 are non-comparable on the edge set E.

CONCLUSION

Now-a-days it becomes fashionable to relate one subject to the other subject. The prime objective of the present work is to develop an idea of the relation between graph-theory and pure mathematics. We wish to examine the concepts of algebra and analysis of the set of graphs. In our work we find it very easily. All the conditions of algebra and analysis are satisfied on the set of graphs. By the meaning set of graphs we meant only the edge set of a simple graph. We have developed the new approach of algebra and analysis of the set of graphs in a very simple and interesting manner.

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